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Analysis of knuckleball trajectories

Alan M. Nathan^{a,*}

^a*University of Illinois, 1100 W. Green St, Urbana IL 61801, USA*

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Abstract

An analysis of knuckleball trajectories is presented using data from the PITCHf/x video tracking system for pitches thrown in actual MLB games. The data reveal that, contrary to popular belief, knuckleball trajectories are as smooth as those from normal pitches. However, the data also show that the deflection of a knuckleball from a straight-line trajectory is essentially random in both magnitude and direction.

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1. Introduction

The knuckleball is perhaps the most mysterious of baseball pitches. It is thrown at a speed significantly lower than that of other pitches and with very little spin. The lack of spin means that the knuckleball does not experience the Magnus force that is responsible for the movement on normal pitches. Nevertheless there is still considerable movement, so much so that the trajectory seems to be completely unpredictable by anyone—the batter, the catcher, or even the pitcher. Much of what we know about knuckleball trajectories is anecdotal [1]. The common perception is that unlike normal pitches, the knuckleball does not follow a smooth trajectory between pitcher and batter but instead can undergo abrupt changes of direction.

The perception is based in part on the results of wind tunnel studies [2,3] that show significant lateral forces on a non-spinning [2] or slowly spinning [3] baseball, with magnitude and direction that depend critically on the orientation of the seam pattern relative to the air flow. Watts and Sawyer [2] investigated the dependence of these forces on the orientation of the baseball in the so-called four-seam (4S) configuration and found an approximate $\sin(4\theta)$ angular dependence. The physical picture is that the flow of air over a seam triggers a transition from laminar to turbulent flow, resulting in a delay of

* Corresponding author. Tel.: +1-217-333-0965; fax: +1-217-333-1215.
E-mail address: a-nathan@illinois.edu.

boundary-layer separation and a deflecting force in the direction of the seam. If such is the case, then the symmetry of the baseball in the 4S configuration would result in a force profile that repeats every 90° , exactly as observed in the data. This result lends credence to the notion that the seams are responsible for the deflecting forces. Watts and Sawyer also found that the peak lateral force varies approximately as the square of the velocity, with the peak force being approximately half the weight of the ball at 65 mph. As noted in [2], if a knuckleball were thrown with zero spin, then it would experience a constant force leading to a smooth trajectory. They speculate that the key to the erratic behavior is to rotate the ball very slowly--no more than half a revolution between pitcher and batter--so that the lateral forces change in both magnitude and direction during the trajectory. Under these conditions simulations show the possibility of trajectories with several changes of direction [2]. The 4S investigations were recently confirmed in a new wind tunnel study [3], in which the forces in the two-seam (2S) orientation were also investigated. Evidently the 2S configuration is preferred by successful knuckleball pitchers [3].

In the present study, we will investigate the trajectories of knuckleballs directly, using actual data from Major League Baseball (MLB) games. The tracking system used to measure the trajectories is described in Section 2. Our analysis will focus on two primary issues. In Section 3 we investigate quantitatively the smoothness of knuckleball trajectories. In particular we investigate whether or not there are sudden changes in direction. In Section 4, we investigate the movement of a knuckleball, that is, the deviation in magnitude and direction from a trajectory with no lateral forces other than gravity. For both smoothness and movement, we compare with that of normal (i.e., non-knuckleball) pitches. We summarize our results in Section 5.

2. The PITCHf/x Tracking System

PITCHf/x is a video-based tracking system created by Sportvision [4]. It is permanently installed in every MLB stadium and has been used since the start of the 2007 season to track every pitch in every MLB game. The system consists of two 60 Hz cameras mounted high above the playing field with roughly orthogonal optical axes and with fields of view that cover most of the 60.5 ft between the pitching rubber and home plate. Proprietary software is used to identify the pixel coordinates of the baseball in each image in real time, which are then converted to a location in the field coordinate system. The latter has its origin at the corner of home plate; the y axis points toward second base (opposite to the primary direction of the pitch); the z axis points vertically up; and x points to the catcher's right, defining a right-handed coordinate system. The conversion utilizes the transformation matrix for each camera [5], which is determined separately using markers placed at precisely known locations on the field. Depending on the details of each installation, the pitch is typically tracked in the approximate range $y=5$ -50 ft, resulting in about 20 images per camera. For the present analysis, each camera image determines a line of position (LOP) connecting the camera to the ball. The intersection of each pair of images, one from each camera, determines the position of the baseball in the field coordinate system to a precision of typically 0.5 inch. Under normal operation, each trajectory is fitted using a constant-acceleration model, so that nine parameters (9P) determine the full trajectory: an initial position, an initial velocity, and an acceleration for each of three coordinates. Simulations have shown that such a parametrization is an excellent description of trajectories for normal pitches. Using the 9P fit, various quantities of interest to both physicists (e.g, drag and lift coefficients) and baseball analysts (e.g., release speed, home plate location, movement) can be calculated. For present purposes, the issue to be addressed is the extent to which a smooth parametrization is an adequate description of knuckleball pitches.

3. Are Knuckleball Trajectories Smooth?

To investigate this question, raw tracking data were obtained from Sportvision for four different games from the 2011 MLB season, two each involving knuckleball pitchers R. A. Dickey of the New York Mets and Tim Wakefield of the Boston Red Sox. Here I will give a detailed analysis for the Florida at New York game on August 29 in which 278 pitches were tracked over the region $y=7-45$ ft, of which 77 were knuckleballs thrown by Dickey. For each pitch, the trajectory was fitted to a more realistic 9-parameter function that still describes a smooth trajectory:

$$\ddot{\mathbf{r}} = -KC_D v^2 \hat{\mathbf{v}} + KC_L v^2 (\hat{\boldsymbol{\omega}} \times \hat{\mathbf{v}}) + \mathbf{g} \quad (1)$$

where K is a numerical factor involving the mass and radius of the ball and the air density, and C_D is the drag coefficient. The first term is the acceleration due to the drag force, the second due to the lateral force, and the third due to gravity. For a normal pitch the second term is due to the Magnus force, with the unit vector $\boldsymbol{\omega}$ pointing along the spin axis and C_L being the lift coefficient. For a knuckleball C_L is a constant that is proportional to the average strength of the lateral force over the full trajectory, and $\boldsymbol{\omega}$ points along a direction orthogonal to both the velocity and the average lateral force. With constant C_L and $\boldsymbol{\omega}$, Eq. 1 necessarily describes a smooth trajectory. On the other hand, the wind tunnel data [2,3] predict variation in both C_L and $\boldsymbol{\omega}$, depending on how the seam orientation changes over the trajectory. Therefore, the analysis will seek to investigate deviations of actual trajectories from those described by Eq. 1. Altogether there are nine fitted parameters that are determined using the Levenberg-Marquardt nonlinear least-squares fitting algorithm along with a fourth-order Runge-Kutta algorithm for numerical integration of the equations of motion [6]: an initial position and velocity for each of the three coordinates, C_D , C_L , and an angle characterizing the direction of $\boldsymbol{\omega}$.

The smoothness of each trajectory will be characterized by the root-mean-square (rms) deviation of the data from the fit. In Fig. 1, the rms value for each pitch is plotted as a function of the percentage of pitches in the sample having a smaller rms value. Normal pitches (blue) and knuckleballs (red) are plotted separately. The nonlinearity of the horizontal axis is such that samples following a Gaussian distribution appear as straight lines, with the central value at 50% and standard deviation equal to the half the difference between the 84% and 16% values. The distributions are very similar, both approximately Gaussian with the same standard deviation (0.04 inch), but with the mean value for knuckleballs (0.326 inch) slightly larger than that for normal pitches (0.295 inch). The deviation of the data from the fit is remarkably small for both types of pitches, indicating that the precision of the tracking data is of order 0.3 inch for each measurement of position. This precision is better than previously thought and corresponds to approximately 0.3 pixels of random noise. This measurement precision is remarkable considering it is about one-tenth the diameter of the ball. Moreover, there is little or no evidence in the data for any significant difference in smoothness between knuckleball and normal trajectories. An example of a knuckleball trajectory is shown in Fig. 2 along with the residuals from the fit. For this particular pitch, the rms was on the high end of the distribution, 0.388 inch, exceeded by only five other knuckleballs pitched in that game. The release speed was 77 mph, which is typical of Dickey's knuckleball. The largest residual was only 0.5 inch and there is nothing to suggest that that deviation is due to anything other than random error in the measurement. We conclude that within the precision of the tracking data (≤ 0.5 inch), knuckleball trajectories are as smooth as those of normal pitches. The identical conclusion applies to pitches from the other three games that were analyzed.

4. The Movement of a Knuckleball

We next investigate the movement of knuckleballs. We utilize a more conventional analysis [7] in which the constant acceleration fit is used to calculate the movement, which is defined to be the deviation of the trajectory from one for which there are no lateral forces other than gravity. For normal pitches, the movement is due to the Magnus force. The 9P fit is used to extrapolate the trajectory to $y=50$ ft and to $y=1.416$ ft, the latter being the front edge of home plate. The movement in both the x (horizontal) and z (vertical) directions are computed between those limits. Approximately 2200 and 2600 pitches thrown by Tim Wakefield and R. A. Dickey, respectively, during the 2010 season were analyzed. To compare with a non-knuckleball pitcher, pitches

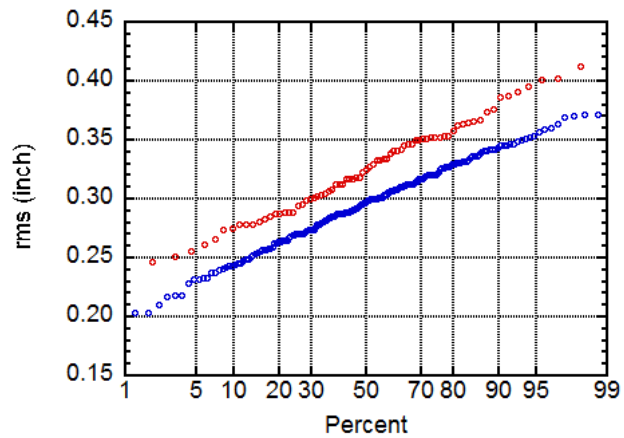


Figure 1 Distribution of rms values for normal (blue) and knuckleball (red) pitches. The nearly parallel linear contours of these samples indicate that they each have a Gaussian distribution with the same standard deviation but with the mean value slightly larger for knuckleballs.

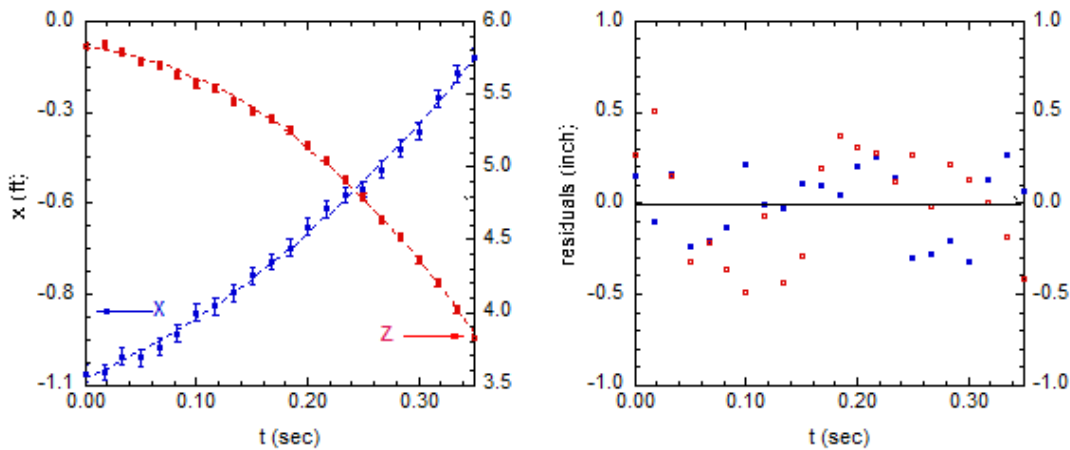


Figure 2. Trajectory of a knuckleball pitch (left) along with the fit. The error flags are 0.025 ft. The right plot shows the difference between the actual and fitted values. The rms deviation of the fit from the data is 0.39 inch, demonstrating the smoothness of the trajectory.

from Boston Red Sox left-hander Jon Lester were also analyzed. The comparison between Lester and Wakefield is given in Fig. 3, which is a polar scatter plot in which the radial coordinate is the release speed and the polar coordinate is the angle of the movement in the x - z plane, with 0° and 90° corresponding to the catcher’s right and up, respectively. For Lester, as with all normal pitchers [7], the plot shows distinct clusters for each of the five pitches in his repertoire. For example, the cluster near 70°

and in the 90-95 mph range is his four-seam fastball, whereas the cluster near 225° and 70-75 mph is his curveball. Now compare with Wakefield, who throws an occasional fastball in the low 70's and $\sim 120^{\circ}$ and a curveball at 60 mph and 315° , but who primarily throws the knuckleball, the latter shown as the nearly uniform ring at about 66 mph. Whereas Lester's pitches have predictable movement, Wakefield's knuckleball does not. Although not presented here, Dickey's knuckleball shows similar behavior to that of Wakefield. The randomness of the knuckleball movement was first noted by Walsh [8].

Fig. 4 is a scatter plot of total movement versus release speed for all of Lester's pitches and for only the knuckleballs for Dickey and Wakefield. Whereas Wakefield throws at a very consistent 66-67 mph, Dickey throw at two speeds: one in the 73-75 mph range, the other in the 75-80 mph range. The plot shows that the movement on the knuckleball is as random in magnitude as it is in direction. Moreover, the maximum movement appears to decrease with increasing speed. Whether that is due to the velocity dependence of the knuckleball forces or due to technique is not possible to say from this analysis. The wind tunnel experiments predict movement that is approximately independent of velocity [2].

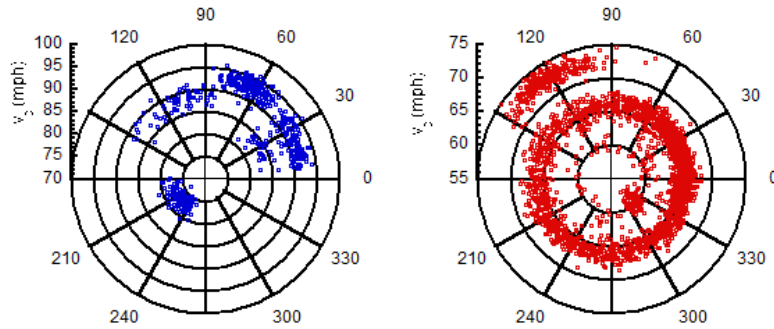


Figure 3. Polar scatter plot of movement direction in the x-z plane versus release speed for normal pitcher Jon Lester (left) and knuckleball pitcher Tim Wakefield (right). The knuckleballs are in the ring centered at 66 mph. Whereas the normal pitches are clustered in angle and speed, the direction of the knuckleball movement appears to be completely random.

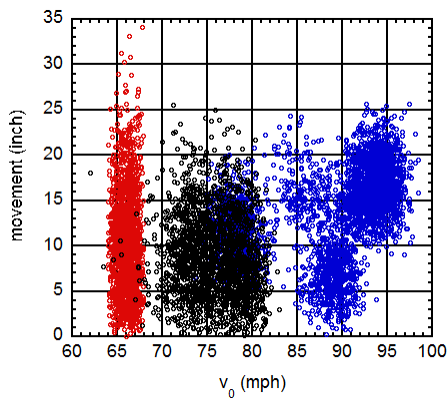


Figure 4 Scatter plot of total movement versus release speed for Wakefield (red), Dickey (black), and Lester (blue). This plot shows that the knuckleball movement is as random in magnitude as it is in direction.

5. Summary and Conclusions

Two principal conclusions result from our study of knuckleball trajectories:

- Knuckleballs follow a trajectory that is as smooth as that of normal pitches, within the ~ 0.5 inch precision of the tracking data.
- Unlike normal pitches, the movement of a knuckleball is random in both magnitude and direction.

The smoothness conclusion appears to contradict the popular belief that knuckleball trajectories are erratic and often experience abrupt changes of direction. However, it is possible that this belief is the result of the randomness of movement, giving rising to a perception of erratic behavior. No attempt was made in the analysis to reconcile our conclusions with the wind tunnel experiments. However, from a purely physics point of view, the trajectory of a

baseball traveling at typical pitched ball speeds cannot make sudden or erratic changes in direction without enormous forces, therefore casting doubt on the popular belief. It is satisfying that this same conclusion is reached by a careful analysis of the data.

The picture that emerges from the trajectory analysis is that a knuckleball trajectory is an example of a chaotic system. That is, small changes in the initial conditions (e.g., seam orientation, rotation rate, or rotation axis) give rise to large changes in the average lateral force on the baseball, resulting in approximately random movement.

Acknowledgements

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